Neutrino oscillations and the effect of the finite lifetime of the neutrino source

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We consider a neutrino source at rest and discuss a condition for the existence of neutrino oscillations which derives from the finite lifetime τ_S of the neutrino source particle. This condition is present if the neutrino source is a free particle such that its wave function is nonstationary. For a Gaussian wave function and with some simplifying assumptions, we study the modification of the usual oscillation probability stemming from τ_S. In the present accelerator experiments the effect of τ_S can be neglected. We discuss some experimental situations where the source lifetime becomes relevant in the oscillation formula.

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I. INTRODUCTION

Neutrino physics is one of the most active fields in particle physics nowadays. Apart from the impressive results of the underground experiments concerning atmospheric and solar neutrino measurements, also reactor and accelerator physics have a large share in the evolution of this field and the understanding of the neutrino mass spectrum and mixing matrix (for recent reviews see, e.g., Refs. [1,2]). Since the phenomenon of neutrino oscillations is now close to being an established physical reality and constitutes the most tangible window for physics beyond the standard model, it is very important to know if there are any limitations to the validity of the usual formula for neutrino survival and transition probabilities [3] with which the experimental results are evaluated. Such questions have been discussed extensively in the literature in the context of the wave packet and field-theoretical approaches (see, e.g., Ref. [4] for a list of references).

In this paper we use the field-theoretical approach in the spirit of Ref. [5], which has proved to be a general and unambiguous method to analyze neutrino oscillations, and concentrate on the effect of the finite lifetime of the neutrino source particle. We study the problem under the following three assumptions: (i) the neutrino source is at rest; (ii) the source particle is not in a bound state and, therefore, is described by a nonstationary wave function; (iii) the wave function of the detector particle is stationary. The first assumption is of a technical nature and allows us to use the methods and results of Refs. [6,7] and can probably be overcome [8]; however, the second one is essential to the discussion in this paper. The third assumption simply means that the particle with which the neutrino reacts in the detector is in a bound state and the wave function of the detector particle does not spread with time [7]. Note that we also neglect, for both the production and the detection process, any possible interaction with the background.

The finite lifetime $\tau_S = 1/\Gamma$ of the neutrino source particle has an impact on the neutrino oscillation probability in the following two ways: (a) a suppression of the probability amplitude as a function of Γ [7], an effect independent of L, the distance between neutrino source and detection; (b) a damping of neutrino oscillations through the coherence length $L_\Gamma^{\rm coh}$ caused by the finite lifetime and given by the factor $\exp(-L/L_\Gamma^{\rm coh})$ [7] in the oscillation probability. The main topic of this paper is the derivation of a correction to the usual neutrino oscillation probability according to point (a) and a numerical study of this correction. It has been shown in Ref. [7] that the effect (a) of Γ is only present provided assumption (ii) is valid. This seems to be the case in the important experiments of the Liquid Scintillation Neutrino Detector (LSND) [9] and KARMEN [10] Collaborations, which study $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations with μ^+ decay at rest[1] as $\bar{\nu}_\mu$ source.[2] In the following we will use the source and detection reactions of LSND and KARMEN as a model for our field-theoretical treatment of neutrino oscillations.

The paper is organized as follows. In Sec. II we derive the transition probability for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations and show how the finite source lifetime changes the standard formula of neutrino oscillations and gives rise to a suppression factor for the interference terms in the oscillation formula. We will see that the corresponding condition for the existence of neutrino oscillations, i.e., the condition that the suppression does not take place, depends only on parameters supplied, at least in principle, by the experimental setup and the neutrino mass squared difference Δm^2. This is natural within the field-theoretical treatment which enables the study of the dependence on those quantities which are really observed or manipulated in oscillation experiments. In Sec. III we apply the results of Sec. II to the case of a Gaussian μ^+ wave function and make a numerical study of the 2-flavor transition probability to investigate quantitatively the influence of the cor-

[1]Note that though the neutrino source particle is not in a bound state in these experiments it is not in a completely freely spreading state either because of the background matter. However, since we neglect the effect of the background matter we work with the simplification of a free time evolution of the muon.

[2]In the LSND experiment also $\nu_\mu \to \nu_e$ oscillations are investigated where the muon neutrinos originate from π^+ decay in flight. According to assumption (i) we do not discuss this case here.

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rection factor. In Sec. IV we present a discussion of our findings and draw the conclusions.

II. THE CROSS SECTION

As mentioned in the Introduction, for definiteness and also for comparison with the notation in Ref. [7] we discuss the process

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \overset{\nu\ \text{osc.}}{\rightsquigarrow} \bar{\nu}_e + p \to n + e^+, \tag{2.1}$$

which is investigated in the LSND and KARMEN experiments. As shown in Ref. [7], with the help of the Weisskopf-Wigner approximation one can write the amplitude of the process (2.1) in the limit $t\to\infty$ as

$$\begin{aligned}\mathcal{A} = (-i)^2 \Big\langle \nu_e(p'_\nu), e_S^+(p'_{eS}); e_D^+(p'_{eD}), n(p'_n) \Big| T \\ \times \Big[\int_0^\infty dt_1 \int d^3x_1 \int_0^\infty dt_2 \int d^3x_2\, \mathcal{H}^+_{S,\text{int}}(x_1) \\ \times e^{-(1/2)\Gamma t_1} \mathcal{H}^+_{D,\text{int}}(x_2) \Big] \Big| \mu^+; p \Big\rangle,\end{aligned} \tag{2.2}$$

where T is the time-ordering symbol and Γ is the total decay width of the muon. $\mathcal{H}^+_{S,\text{int}}$ and $\mathcal{H}^+_{D,\text{int}}$ are the relevant Hamiltonian densities (in the interaction picture) for the production and the detection of the neutrinos, describing muon decay and proton to neutron transition (2.1), respectively. The indices S and D denote source and detection, respectively. The muon μ^+ and the proton p are localized at the coordinates $\vec{x}_S$ and $\vec{x}_D$, respectively. The proton state is stationary whereas the decaying muon at rest is described by a free wave packet with an average momentum equal to zero according to the assumptions (i)–(iii). Hence the spinors of the initial particles are written as

$$\begin{aligned}&\Psi_p(x) = \psi_p(\vec{x}-\vec{x}_D) e^{-iE_p t} \\ &\quad \text{with}\ \ \psi_p(\vec{x}-\vec{x}_D) = \frac{1}{(2\pi)^{3/2}} \int d^3p\, \tilde{\psi}_p(\vec{p}) e^{i\vec{p}\cdot(\vec{x}-\vec{x}_D)} \\ &\quad \text{and}\ \ \tilde{\psi}_p(\vec{p}) = \tilde{\psi}'_p(\vec{p}) u_p(\vec{p})\end{aligned} \tag{2.3}$$

for the proton and

$$\begin{aligned}&\psi_\mu(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \tilde{\psi}_\mu(\vec{p}) e^{-i(\vec{p}\cdot\vec{x} - E_\mu(\vec{p})t)} e^{i\vec{p}\cdot\vec{x}_S} \\ &\text{with}\ \ \tilde{\psi}_\mu(\vec{p}) = \tilde{\psi}'_\mu(\vec{p}) v_\mu(\vec{p})\end{aligned} \tag{2.4}$$

for the muon with $E_\mu(\vec{p}) = \sqrt{m_\mu^2 + \vec{p}^{\,2}}$. In the 4-spinors $u_p(\vec{p})$ and $v_\mu(\vec{p})$ we have left out the polarizations of the proton and muon, respectively, because they are irrelevant in the further discussion.

The function $\psi_p(\vec{y})$ is peaked at $\vec{y}=\vec{0}$ and the wave packet $\tilde{\psi}_\mu(\vec{p})$ in momentum space is peaked around the average momentum $\langle\vec{p}\rangle = \vec{0}$. The final particles are described by plane waves. After carrying out all integrations, the leading term of the amplitude in the asymptotic limit $L\to\infty$ can be written as [7]

$$\mathcal{A}^\infty = \sum_j U_{\mu j} U^*_{ej} e^{iq_j L} \mathcal{A}^S_j \mathcal{A}^D_j, \tag{2.5}$$

where $\mathcal{A}^S_j$ and $\mathcal{A}^D_j$ denote the amplitudes for production and detection (2.1) of a neutrino with mass m_j, respectively. The parts of these amplitudes which are important for the further discussion are given by

$$\mathcal{A}^S_j = \frac{1}{i(E_{Sj} - E_D) + \frac{1}{2}\Gamma} \bar{\tilde{\psi}}_\mu(\vec{p}_1 + q_j \vec{l}) \cdots \tag{2.6}$$

and

$$\mathcal{A}^D_j = \cdots \tilde{\psi}_p(-q_j \vec{l} + \vec{p}_2). \tag{2.7}$$

For the full expressions see Ref. [7]. The kinematical quantities occurring in Eqs. (2.6) and (2.7) are defined by

$$\begin{aligned}&q_j = \sqrt{E_D^2 - m_j^2}, \\ &E_D = E'_n + E'_{eD} - E_p, \\ &E_{Sj} = E_\mu(q_j \vec{l} + \vec{p}_1) - E'_\nu - E'_{eS}\end{aligned} \tag{2.8}$$

and

$$\begin{aligned}&\vec{p}_1 = \vec{p}'_\nu + \vec{p}'_{eS}, \quad \vec{p}_2 = \vec{p}'_n + \vec{p}'_{eD}, \quad L = |\vec{x}_D - \vec{x}_S|\ \ \text{and} \\ &\qquad \vec{l} = (\vec{x}_D - \vec{x}_S)/L.\end{aligned} \tag{2.9}$$

Note that the first two formulas in Eq. (2.8) follow from assumption (iii) in the introduction [6,7]. Equation (2.6) shows the dependence of $\mathcal{A}^\infty$ on Γ [see point (a) in the Introduction]. The structure (2.5) arises from the fact that in the limit $L\to\infty$ the neutrinos with mass m_j, described by an inner line of the Feynman diagram derived from $\mathcal{A}$ (2.2), are on mass shell. Consequently, the amplitude for neutrino production and its subsequent detection factorizes for each j into a product of production amplitude and detection amplitude [6]. Looking at Eqs. (2.5), (2.6) and (2.7) it is evident that oscillations involving $m_j^2 - m_k^2$ can only take place if the conditions [5,6]

$$|q_j - q_k| \lesssim \sigma_S \ \ \text{and}\ \ |q_j - q_k| \lesssim \sigma_D \tag{2.10}$$

and [7]

$$|E_{Sj} - E_{Sk}| \lesssim \frac{1}{2}\Gamma \tag{2.11}$$

hold, where σ_S and σ_D are the widths of $\tilde{\psi}_\mu$ and $\tilde{\psi}_p$, respectively. We call conditions (2.10) amplitude coherence conditions (ACC) and Eq. (2.11) the source wave packet–finite

lifetime condition (SFC) [7]. If they are not satisfied then some terms of the amplitude and consequently the corresponding interference terms in the cross section (oscillation probability) are suppressed.

The cross section σ is obtained by taking the absolute square of the amplitude (2.5), integrating over the final state momenta $\vec{p}'_\nu$, $\vec{p}'_{eS}$, $\vec{p}'_n$, $\vec{p}'_{eD}$ and averaging over the spins of the initial muon and proton. Hence

$$\sigma=\int dP|\mathcal{A}^\infty|^2, \tag{2.12}$$

where dP denotes the integration over the momenta of the particles at the external legs and is given by

$$dP=\frac{d^3p'_\nu}{2E'_\nu}\frac{d^3p'_{eS}}{2E'_{eS}}\frac{d^3p'_n}{2E'_n}\frac{d^3p'_{eD}}{2E'_{eD}} \tag{2.13}$$

and the integration is done over some volume of the phase space.

In general, the integrations in Eq. (2.12) cannot be performed without knowledge of the source and detector wave functions. However, as noticed in Ref. [7], for

$$\Gamma\ll\sigma_{S,D} \tag{2.14}$$

the factors

$$\{(i(E_{Sj}-E_D)+\Gamma/2)\times(-i(E_{Sk}-E_D)+\Gamma/2)\}^{-1} \tag{2.15}$$

in the cross section are strongly peaked with respect to E_D and we interpret the conditions (2.14) that the rest of the cross section is flat with respect to E_D if varied over intervals several orders of magnitude larger than Γ. This is a reasonable assumption because in the LSND and KARMEN experiments the stopped muons have momenta of the order 0.01 MeV [11,12], and thus σ_S will be in the same range. Assuming atomic dimensions of the spread of the detector particle wave function, we find $\sigma_D\sim10^{-3}$ MeV. In any case, even if our guesses for σ_S and σ_D are wrong by several orders of magnitude, these widths can never be so small like the decay constant of the muon $\Gamma\simeq3\times10^{-16}$ MeV. Therefore, we adopt the procedure that integrating over momenta of the final state of the detector leads to an integration in the variable E_D and in view of the expressions (2.15) it suffices to pick out an integration interval of length ΔE_D which satisfies $\Gamma\ll\Delta E_D\ll\sigma_{S,D}$. Considering neutrino oscillations with respect to $m_j^2-m_k^2\equiv\Delta m_{jk}^2$, then this integration interval should contain E_{Sj} and E_{Sk}. Since the condition (2.11) is necessary for these oscillations to happen at all, we define a mean value

$$\overline{E}_D\equiv\langle E_{Sj}\rangle \tag{2.16}$$

for the neutrino mass eigenfields participating in the oscillations and ΔE_D only has to be a few orders of magnitude larger than Γ. In addition, the expressions (2.15) suggest to close the integration over the interval of length ΔE_D along the real axis by a half circle in the complex plane and to apply Cauchy's theorem [7].

In this way we have shown that the cross section contains the damping factor [7]

$$\exp\left(-\frac{\Delta m_{jk}^2\Gamma}{4\overline{E}_D^2}L\right) \tag{2.17}$$

leading to a suppression of the corresponding interference term in the oscillation probability when L becomes greater than the coherence length $L_\Gamma^{\rm coh}=4\overline{E}_D^2/\Delta m_{jk}^2\Gamma$. As already discussed in Ref. [7], this coherence length is of the order of 100 light years for typical input relevant for the LSND and KARMEN experiments and the suppression due to such a long coherence length is certainly negligible for all experiments with terrestrial neutrinos. In the following computations we will neglect the coherence length arising from the finite lifetime of the neutrino source.

Then, the only relevant integrand with respect to E_D is given by the factors (2.15) and we get

$$\int_{-\infty}^{\infty}dE_D\frac{1}{i(E_{Sj}-E_D)+\frac{1}{2}\Gamma}\frac{1}{-i(E_{Sk}-E_D)+\frac{1}{2}\Gamma}$$
$$=\frac{2\pi}{\Gamma}\frac{1}{1+i\frac{1}{\Gamma}(E_{Sj}-E_{Sk})}. \tag{2.18}$$

On the right-hand side of this equation we can use assumption (i) (the muon at rest) to compute to a very good approximation the energy difference

$$E_{Sj}-E_{Sk}\simeq-\frac{\Delta m_{jk}^2}{2m_\mu\overline{E}_D}\vec{l}\cdot(\vec{p}_1+\overline{E}_D\vec{l}). \tag{2.19}$$

As the next step, we want to perform the integration over $d^3p'_\nu$. Note that the momentum $\vec{p}'_\nu$ of the ν_e from the source reaction [see Eq. (2.1)] cannot be measured. Again we make a simplifying approximation based on assumption (i), namely that the muon wave function can be represented by

$$\tilde{\psi}_\mu(\vec{p}_1+q_j\vec{l})\simeq\tilde{\psi}'_\mu(\vec{p}_1+\overline{E}_D\vec{l})v_\mu(\vec{0}). \tag{2.20}$$

Then the integration over $d^3p'_\nu$ concerns only the functions $(E_{Sj}-E_{Sk})$ in Eq. (2.18) and $\tilde{\psi}'_\mu$. Therefore, we define the quantities

$$g_{jk}=\int d^3u|\tilde{\psi}'_\mu(\vec{u})|^2\frac{1}{1-i\rho_{jk}\vec{l}\cdot\vec{u}/\sigma_S}, \tag{2.21}$$

where we have defined $\vec{u}\equiv\vec{p}_1+E_\nu\vec{l}$ and

$$\rho_{jk}\equiv\frac{\Delta m_{jk}^2\sigma_S}{2m_\mu E_\nu\Gamma}. \tag{2.22}$$

Note that in view of Eqs. (2.8) and (2.16) we identify the neutrino energy which can in principle be measured by the detector as

$$E_\nu \equiv \overline{E_D}. \tag{2.23}$$

The quantities (2.21) have the properties that $g_{jk}=g_{kj}^*$ and $g_{jj}=1$. Since we consider ultrarelativistic neutrinos, we can take the limit $m_j \to 0 \ \forall j$ in all terms of the cross section (2.12) except the oscillation phases and g_{jk}. Then the probability for $\overline{\nu_\mu} \to \overline{\nu_e}$ oscillations is given by

$$P_{\overline{\nu_\mu}\to\overline{\nu_e}} = \sum_j |U_{ej}|^2 |U_{\mu j}|^2 + 2\,\mathrm{Re}\left\{ \sum_{j>k} U_{ej}^* U_{\mu j} U_{ek} U_{\mu k}^* g_{jk} \exp\left(-i \frac{\Delta m_{jk}^2 L}{2E_\nu} \right) \right\}. \tag{2.24}$$

In contrast to the formula derived within the standard treatment of neutrino oscillations, the quantities g_{jk} appear as correction factors in Eq. (2.24). These nonstandard factors g_{jk} represent the quantitative effect of the SFC condition (2.11) discussed in Ref. [7]. They arise from assumption (ii) (see Introduction) that the muon is described by a free wave function, which does not have a sharp energy, and would disappear for a muon in a stationary (bound) state (see Ref. [6]).

III. THE CASE OF A GAUSSIAN MUON WAVE PACKET

In order to get a feeling for the effect of the g-factors (2.21) on the oscillation probabilities we confine ourselves now to the case of a Gaussian muon wave function

$$\tilde{\psi}'_\mu(\vec{p}) = (\sqrt{\pi}\sigma_S)^{-3/2} \exp\left(-\frac{\vec{p}^2}{2\sigma_S^2} \right). \tag{3.1}$$

Then we get

$$g_{jk} = \frac{1}{\sqrt{\pi}\sigma_S} \int_{-\infty}^{\infty} du\, e^{-u^2/\sigma_S^2} \frac{1}{1 - i\rho_{jk} u/\sigma_S}, \tag{3.2}$$

where $u \equiv \vec{l} \cdot \vec{u}$. Since the exponential factor in Eq. (3.2) is an even function with respect to the variable u the g-factors become real and defining $y = u/\sigma_S$ one obtains $g_{jk} = g(\rho_{jk})$ with

$$g(\rho) \equiv \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dy \frac{e^{-y^2}}{1+\rho^2 y^2} = \frac{\sqrt{\pi}}{\rho} \exp\left(\frac{1}{\rho^2} \right) \Phi_c\left(\frac{1}{\rho} \right), \tag{3.3}$$

where Φ_c is the complementary error function (see, e.g., Ref. [15]). Note that $g(\rho) \simeq 1 - \rho^2/2$ for $\rho \ll 1$ and $g(\rho) \to \sqrt{\pi}/\rho$ for $\rho \to \infty$. Under the assumption of oscillations between two neutrino flavors the transition probability becomes

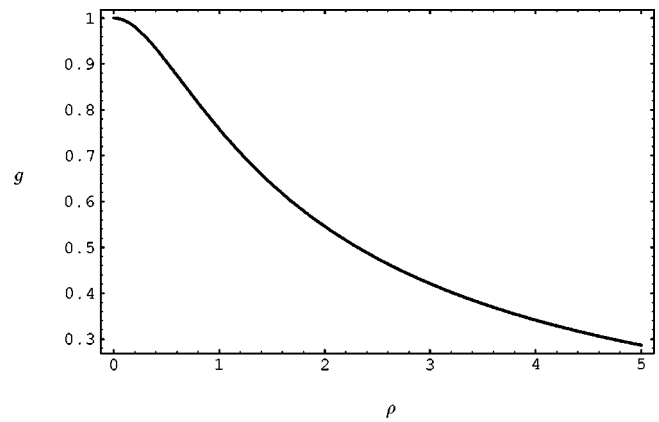

FIG. 1. Plot of the correction factor g [Eq. (3.3)] as a function of ρ.

$$P_{\overline{\nu_\mu}\to\overline{\nu_e}} = \frac{1}{2} \sin^2 2\theta \left(1 - g_{12} \cos \frac{\Delta m^2 L}{2E_\nu} \right), \tag{3.4}$$

where θ is the 2-flavor mixing angle. In comparison with the standard formula the factor g_{12} in Eq. (3.4) lowers for a given transition probability the upper bounds on Δm^2 and $\sin^2 2\theta$. This can be seen explicitly by considering the low Δm^2 region of the parameter space where the cosine in expression (3.4) can be expanded in Δm^2. If now $\rho \lesssim 1$ holds for an average Δm^2, in the low Δm^2 region we get $\rho \ll 1$ and can use the corresponding expansion of $g(\rho)$ given above. Then, in this region the isoprobability contour is determined by the relation

$$(\Delta m^2)^2 \sin^2 2\theta \simeq \frac{16 E_\nu^2 P_{\overline{\nu_\mu}\to\overline{\nu_e}}}{L^2 + \left(\frac{\sigma_S \tau_S}{m_\mu} \right)^2}. \tag{3.5}$$

It is clear from Eq. (3.5) that a large source lifetime and a large source momentum spread lower the upper bounds on Δm^2 and $\sin^2 2\theta$ compared to the standard oscillation formula. For the source lifetime effect to be significant in a given experiment, the numerical value of $\sigma_S \tau_S / m_\mu$ must be comparable to the source detector distance L.

In the following we investigate quantitatively the effect of g_{12} on the transition probability (3.4). In Fig. 1 we display the function $g(\rho)$. One can see that at $\rho = 0.5$ the function g is around 0.9 which means that g_{12} starts to deviate appreciably from 1 when ρ becomes greater than about 0.5. In the case of the LSND and KARMEN experiments, using $\Gamma \simeq 3 \times 10^{-16}$ MeV for the decay width of the muon and the typical numbers $\sigma_S \simeq 0.01$ MeV [11,12], $\Delta m^2 \simeq 1$ eV^2 and $E_\nu \simeq 30$ MeV, we obtain $\rho_{12} \simeq 0.5 \times 10^{-2}$. From this estimate we conclude that with the above input numbers the correction to the transition probability is negligible for the LSND and KARMEN experiments because $1 - g_{12} \sim 10^{-5}$ [see remark after definition of g (3.3)]. Note, however, that $\rho_{12} \simeq 0.5 \times 10^{-2}$ is only two orders of magnitude away from having a 10% effect. This motivates us to have a look at the influence of ρ_{12} (g_{12}) on exclusion curves obtained, for instance, in the KARMEN experiment. Therefore, we use L

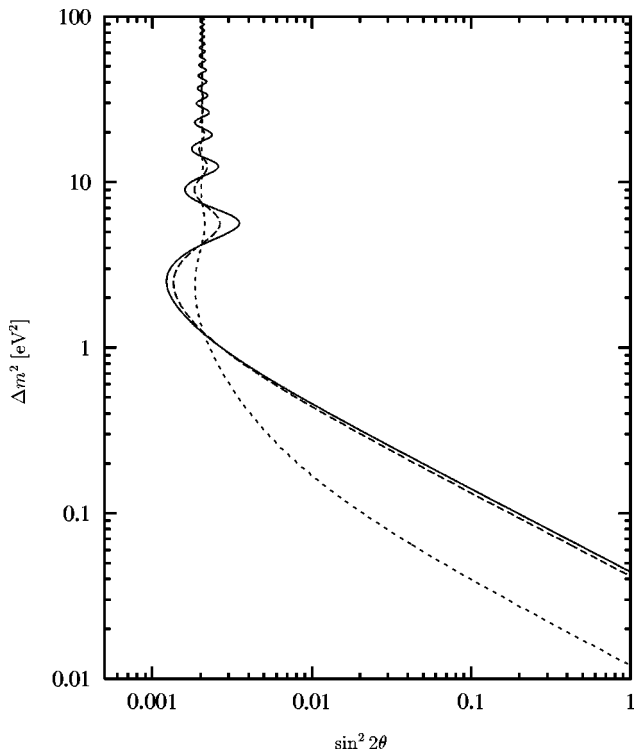

FIG. 2. Contour plot for the transition probability $\overline{P} = \int \phi(E_\nu) P^{-}_{\nu_\mu \to \overline{\nu}_e}(E_\nu) dE_\nu$ at $\overline{P} = 0.001$ in the $\Delta m^2 - \sin^2 2\theta$ plane where $P^{-}_{\nu_\mu \to \overline{\nu}_e}$ is defined in Eq. (3.4) and ϕ denotes the energy distribution of the $\overline{\nu}_\mu$ flux (see, e.g., Ref. [16]). We integrated over the neutrino energy E_ν from 12 up to 53 MeV and used $L = 17.7$ m for the source-detector distance L. The solid, dashed and dotted line correspond to $\sigma_S = 0$ $(g=1)$, 1 and 10 MeV, respectively.

$=17.7$ m, average in Eq. (3.4) over the energy spectrum of $\overline{\nu}_\mu$ and as a gedanken experiment we vary the momentum spread σ_S. In Fig. 2 we show the corresponding exclusion curves in the Δm^2–$\sin^2 2\theta$ plane for the cases of $\sigma_S = 0$, 1 and 10 MeV corresponding to $g_{12} \simeq 1$, 0.9 and 0.3, respectively. We can see that for $\sigma_S = 10$ MeV the exclusion curve has changed noticeably.

IV. DISCUSSION AND CONCLUSIONS

In this paper we have studied the effects of the finite lifetime of the neutrino source particle. In particular, we have elaborated the condition for the existence of neutrino oscillations arising in a situation where the neutrino source particle is not described by a bound state and thus represented by a nonstationary wave function. Furthermore, in our analysis we have made the assumption that the neutrino source is at rest. In order to perform an analysis closely related to experiment we used the field-theoretical approach which allows us to work directly with the states of the particles responsible for neutrino production and detection and where the oscillating neutrinos are represented by an inner line in the complete Feynman diagram containing the source and detection processes. Thus the neutrino oscillation amplitude is determined simply through the neutrino production and detection interaction Hamiltonians. For simplicity, in the following we will concentrate on 2-flavor oscillations.

The finite lifetime of the neutrino source leads to a finite coherence length [13]. In our case we have explicitly indicated the damping factor (2.17) from where the coherence length $L_\Gamma^{\rm coh}$ associated with Γ can be read off. Using numbers taken from the LSND and KARMEN experiments for the quantities appearing in $L_\Gamma^{\rm coh}$ we get a length of the order of 100 light years [7] which is absolutely negligible compared the coherence length stemming from the fact that the neutrino energy E_D [see Eqs. (2.8) and (2.16)] determined by the energies of the final states in the detection process is not accurately known. Therefore, the effect (b) of Γ (see Introduction) is irrelevant.

Let us now compare the amplitude coherence condition (ACC) with the source wave packet—finite lifetime condition (SFC). The ACC can be reformulated as [14,5,6]

$$\sigma_{xS,xD} \lesssim \frac{1}{4\pi} L^{\rm osc}, \tag{4.1}$$

where $\sigma_{xS,xD}$ are the widths of the source and detector wave functions, respectively, in coordinate space and $L^{\rm osc} = 4\pi E_\nu / \Delta m^2 = 2.48{\rm m}(E_\nu/1{\rm MeV})(1{\rm eV}^2/\Delta m^2)$ is the oscillation length. For the LSND and KARMEN experiments with $E_\nu \sim 30$ MeV and $\Delta m^2 \sim 1$ eV^2, as found by the LSND Collaboration, one gets $L^{\rm osc} \sim 75$ m. This is to be compared with the widths $\sigma_{xS,xD}$ which are certainly in the range of atomic distances or smaller. Therefore the conditions (4.1) are very well satisfied. We want to stress once more that the SFC and the correction factor g_{12} (2.21) corresponding to effect (a) of Γ (see Introduction) derive from two ingredients: from the finite lifetime of the muon taken into account through the Weisskopf-Wigner approximation and from assumption (ii) in the Introduction that the wave function of the muon is nonstationary. The first ingredient leads to the factor $1/(i(E_{Sj} - E_D) + \frac{1}{2}\Gamma)$ in the amplitude instead of the familiar delta function. The second one leads to the momentum dependence of E_{Sj} as given in Eq. (2.8). If the neutrino source particle were in a bound state and, therefore, described by a stationary wave function, E_S would be fixed and independent of j and momenta, and there would be no condition (2.11). Note that in our approximation the Hamiltonian which determines the wave function of the source particle does not include weak interactions. However, the decay of the source particle via the weak interactions—whether its wave function is stationary or nonstationary—is explicitly given by the factor $\exp(-\Gamma t_1/2)$ in the amplitude (2.2) according to the Weisskopf–Wigner approximation. As shown in Sec. III the correction factor g_{12} depends on the quantity

$$\rho = \frac{\Delta m^2 \sigma_S}{2 m_\mu E_\nu \Gamma}, \tag{4.2}$$

and is only sizeably different from one if $\rho \gtrsim 1$. Thus a convenient formulation of SFC is $\rho < 1$. Note that by defining a spread in velocity of the source wave packet by $\Delta v_S = \sigma_S / m_\mu$ the last formulation of SFC can be rewritten in analogy to Eq. (4.1) as [7]

$$\Delta v_S \tau_S \lesssim \frac{1}{4\pi} L^{\rm osc}. \tag{4.3}$$

As we have seen in Sec. III with $\sigma_S \sim 0.01$ MeV one gets $\rho \sim 0.005$ and SFC is well fulfilled for the LSND and KARMEN experiments, however, the margin for violation of SFC is only two orders of magnitude, in contrast to ACC where the margin is at least ten orders of magnitude.

Taking this observation as a starting point, we want to see if there is some chance to increase ρ to 1 or more in order to have an observable suppression effect. Looking at Fig. 2 one recognizes that in the case of $\sigma_S = 10$ MeV the lower line of the curve has undergone a considerable shift downwards compared to the case $\rho = 0$. Note that this shift happens in the sensitive region of the LSND and KARMEN experiments. One might therefore ask the question if it is possible to increase σ_S in order to achieve $\rho \gtrsim 1$. If the source wave function is such that $\sigma_S \sigma_{xS} \sim 1$, which is correct for Gaussian wave functions, one would have to localize the neutrino source particle very well in coordinate space, namely to $\sigma_{xS} \lesssim 10^{-12}$ m for $\sigma_S \gtrsim 0.1$ MeV. Taking a look at Eq. (4.2), one observes that ρ can also be increased by decreasing the neutrino energy. However, in this case one lowers the ratio of true over background events. Finally, a small product of mass times decay width of the source particle also enhances ρ but, unfortunately, the muon has already the smallest such product among the particles which can copiously be produced.[3]

Note that from Eq. (4.3) it is obvious that in the LSND and KARMEN experiments we could only have an effect of suppressing neutrino oscillations through the spread of the μ^+ wave function if, during the lifetime of the μ^+, the spread evolves over several meters and, hence, goes far beyond the target where the muons are produced and stopped. We want to remind the reader that we have made the idealization of neglecting the presence of the matter background in the neutrino production and detection processes. Consequently, we have neglected the electromagnetic interactions of the μ^+ with the electrons and nuclei in the target. This puts our consideration to a certain extent on the level of a gedanken experiment. However, we want to stress that the effect of the matter background on the spread of the μ^+ wave function is rather to interrupt this spread. Therefore, since we have shown that observable effects of the finite muon lifetime are absent in the idealized situation, the same is certainly true for the real experiment.

The quantum-mechanical ACC and SFC in the forms (4.1) and (4.3), respectively, have analogous ''classical macroscopic'' conditions stemming from the inescapable averaging over some regions of the target and the detector, respectively, when an experiment is performed. Replacing in Eq. (4.1) the widths $\sigma_{xS,xD}$ by the typical sizes $R_{S,D}$, respectively, of the regions in the target where the muon is stopped and in the detector where the neutrino detection process is localized we arrive at conditions which are obtained by the incoherent averaging of the oscillation probabilities over the variations of L due to $R_{S,D}$ and the requirement that neutrino oscillations should not be washed out by this averaging process. Since $R_{S,D}$ are macroscopic quantities, if these classical conditions are satisfied, then clearly also ACC holds because $\sigma_{xS,xD} \ll R_{S,D}$. The classical analogue to SFC (4.3) says that during its lifetime the neutrino source particle should move a distance much less than the oscillation length in order not to wash out neutrino oscillations. This classical condition is not obviously linked to SFC.

[3]The corresponding product for the neutron is smaller, however, to observe this effect in neutrinos from neutron decay one would have to isolate the neutrons from the external interactions—something that seems to be hard to achieve in experiments.

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